Light Source I

Takashi TANAKA (RIKEN SPring-8 Center)

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Light Source I

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- Overview of SR Light Source
- Characteristics of SR (1)

Light Source II

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- Practical and Useful Knowledge

SPECTRA (Thu. 16:20~)

- Computation of SR: Introduction to “SPECTRA”

Introduction

SR Facility and Light Source

- SR: Definition
  - Electromagnetic wave emitted by a charged particle deflected by a magnetic force
- SR Facility
  - Accelerators to generate a high-energy electron beam
  - **Magnetic devices (SR light source) to generate intense SR**
  - Optical elements (monochromators, mirrors,..)
  - Experimental stations

SR as a Probe for Research

- SR has a lot of advantages over other conventional light sources
  - Highly collimated (laser-like)
  - Wavelength tunability
  - Polarization
  - .....
- However, the total radiation power does not differ significantly.

Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
  - Physical quantity of SR
  - Uncertainty of light: Fourier and diffraction limits
  - SR: Light from a moving electron
- Overview of SR Light Source
  - Types of light sources
  - Magnet material and configuration
- Characteristics of SR (1)
  - Radiation from bending magnets

Topics in This Lecture (2)

- Characteristics of SR (2)
  - Undulator radiation
    - Fundamental energy
    - Spectral function
    - Spatial profile of radiation
    - Higher harmonics
  - Wiggler radiation
    - Qualitative descriptions
- Practical and Useful Knowledge of SR
  - Finite emittance and energy spread
  - Heat load and photon flux
  - Wiggler or Undulator?

Fundamentals of Light and SR

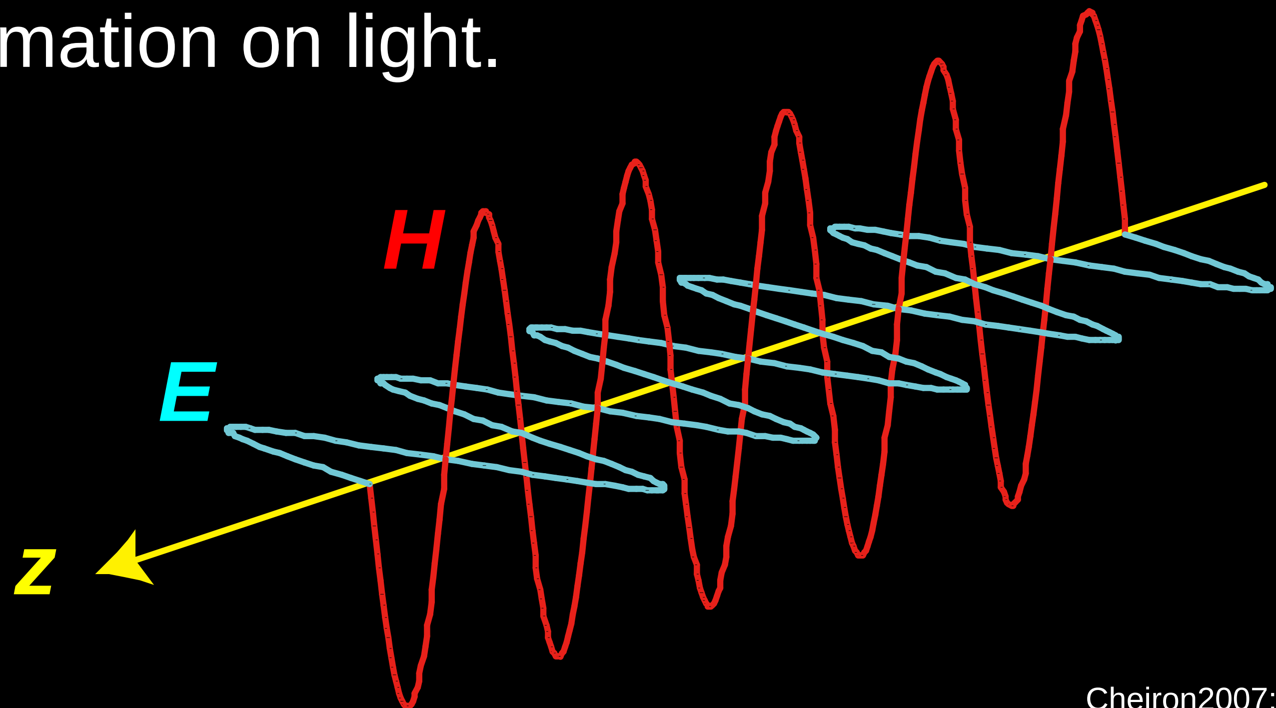
- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

Light as an Electromagnetic Wave

- Electric and magnetic fields can transmit in vacuum as a transverse wave, with a relation

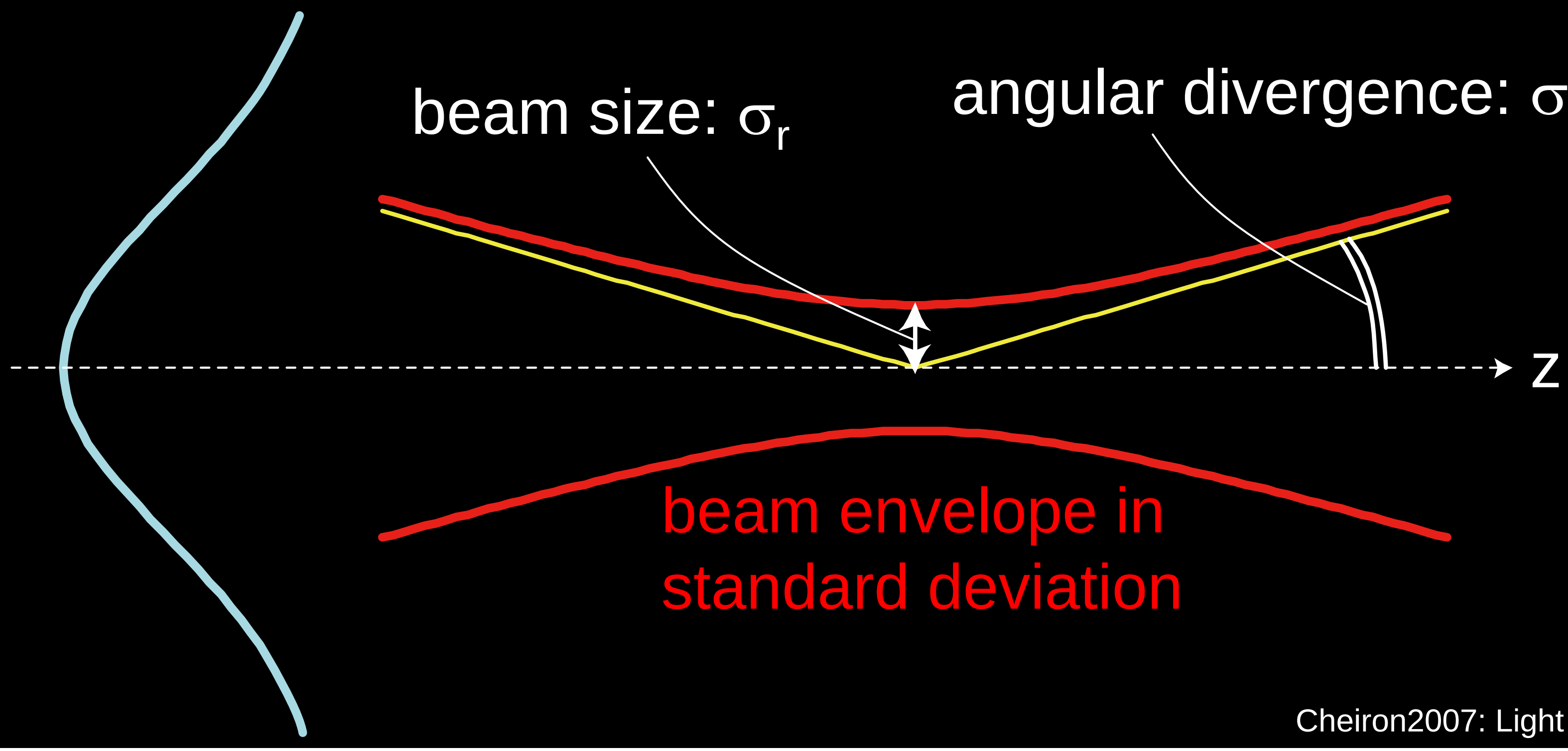
$$\boldsymbol{H} = \frac{1}{\mu_0 c} \hat{\boldsymbol{z}} \times \boldsymbol{E}$$

- Analysis of the electric field gives all the information on light.



Beam Size and Angular Divergence

- Due to diffraction, the transverse profile of light varies as the light propagates.
- For example, the Gaussian beam has a Gaussian transverse profile with a beam envelope varying along the beam axis.



Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

Physical Quantity of Light

- Electric Field of Radiation
  - Time domain
  - Frequency domain

→ Fundamental
Not measurable

- Power Density
- Flux Density

→ Radiation Intensity
Measurable

- (Peak) Brilliance → Universal Figure of Merit
Not Measurable

- Stokes Parameter → Polarization State
Measurable

Electric Field of Radiation

- Electric field of radiation in the time domain, $\boldsymbol{E}(\boldsymbol{r},t)$, generated by a single electron, is given by the Lienard-Wiecherd potential.
- Temporal Fourier transform, $\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega)$, gives the amplitude and phase of an electromagnetic wave at ω and also referred to as complex amplitude.

$$\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega) = \int \boldsymbol{E}(\boldsymbol{r},t)\mathrm{e}^{i\omega t}$$

Radiation Power and Power Density

- Radiation power density indicates the energy density per unit time.
- Radiation power is obtained by integration over a spatial range of interest (e.g., over the slit area, whole solid angle, etc.).

$$\frac{dP}{dS} \propto \int dt |\boldsymbol{E}(\boldsymbol{r}, t)|^2 \quad \text{(W/mm}^2\text{)}$$

$$P \propto \iint dS dt |\boldsymbol{E}(\boldsymbol{r}, t)|^2 \quad \text{(W)}$$

Photon Flux and Flux Density

- Photon flux density indicates the photon density at a given photon energy per unit bandwidth.
- Photon flux is obtained by integration over a spatial range of interest.

$$\frac{d^2F}{dS\,d\omega/\omega} \propto |\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega)|^2$$

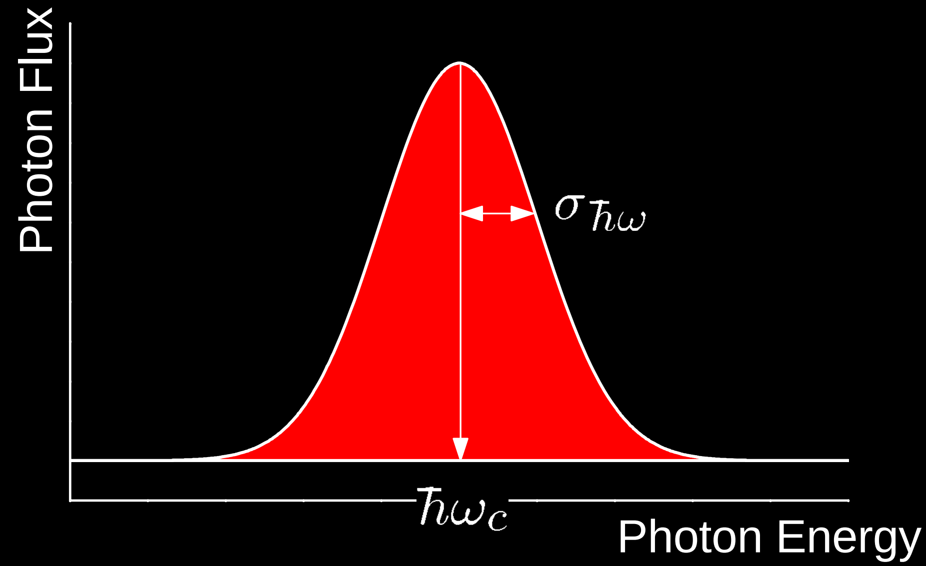
(photons/sec/mm^2 /0.1%b.w.)

$$\frac{dF}{d\omega/\omega} \propto \int dS|\tilde{\boldsymbol{E}}(\boldsymbol{r},\omega)|^2$$

(photons/sec/0.1%b.w.)

Photon Flux and Radiation Power

Radiation power is obtained by integrating the photon flux over the whole energy



$$P = Q_e \int \frac{dF}{d\omega/\omega} \times \hbar\omega \times \frac{d\omega}{10^{-3}\omega}$$

$$= 10^3 Q_e \sqrt{2\pi}\,\sigma_{\hbar\omega} \left[\frac{dF}{d\omega/\omega}\right]_{\omega=\omega_c}$$

(Gaussian Approximation)

Brilliance (Brightness)

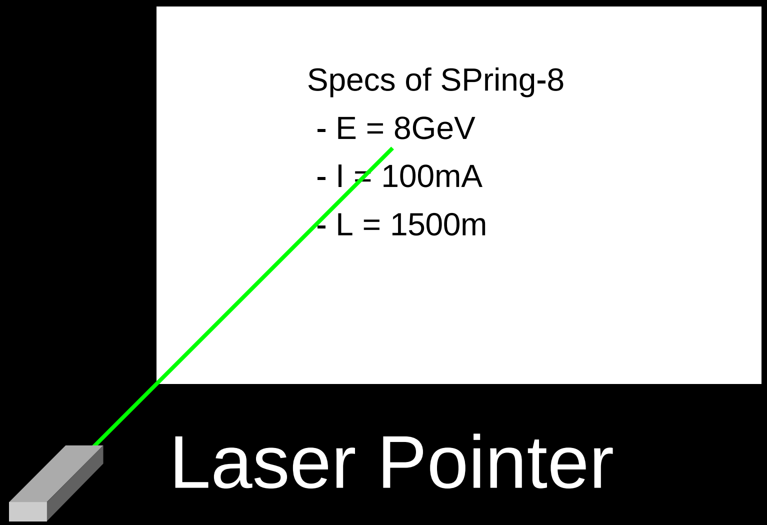
- Strictly speaking, brilliance is defined by the photon flux density in the 4D phase space at a given photon energy.
- For the sake of convenience, brilliance is roughly obtained by dividing the photon flux by the beam size and angular divergence.

$$B = \frac{dF/(d\omega/\omega)}{4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}}$$

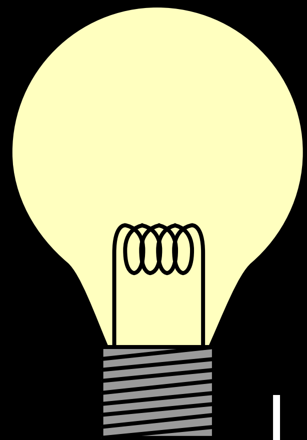
(photons/sec/mrad^2 /mm^2 /0.1%b.w.)

larger than the volume in the 4D phase space defined by the diffraction limit

Example: Laser Pointer and Light Bulb



Laser Pointer



Light Bulb

	Bulb	Laser
$\lambda_r(\hbar\omega_r)$	500nm (2.5eV)	
Total Power: W	100 W	1 mW
Angular Divergence	4π rad	1 mrad
Beam Size:	1 cm^2	1 mm^2
Band Width:	100 (%)	0.01 (%)

Example: Photon Flux

$$\left[\frac{dF}{d\omega/\omega}\right]_{\omega=\omega_r} = \frac{P}{10^3 Q_e \sqrt{2\pi}\sigma_{\hbar\omega}}$$

	Bulb	Laser
Total Power	100 W	1 mW
Band Width	100 (%)	0.01 (%)
$\sigma_{\hbar\omega}$	2.5 eV	2.5x10^{-4} eV
Flux@500nm	10^{17}	10^{16}

Example: Brilliance

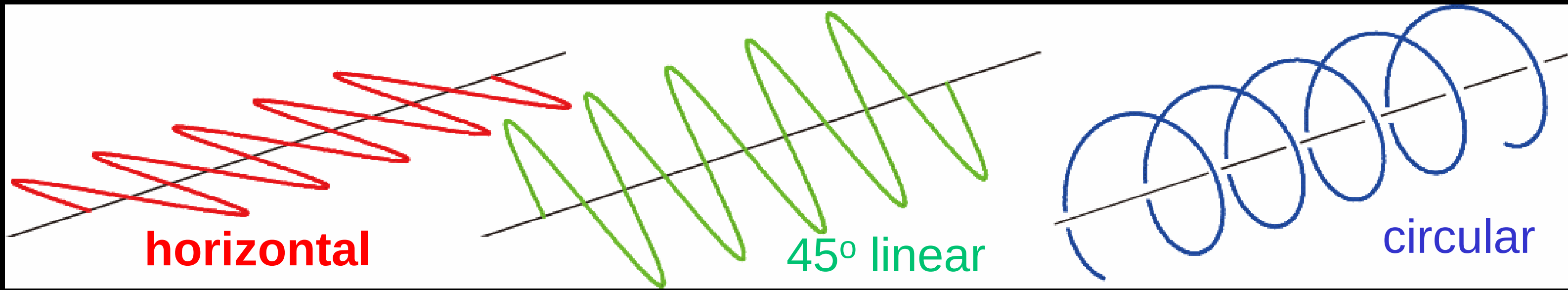
$$B = \frac{dF/(d\omega/\omega)}{4\pi^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'}}$$

	Bulb	Laser
Total Power	100 W	1 mW
Flux@500nm	10^{17}	10^{16}
Angular Divergence	1 rad	1 $mrad^2$
Beam Size	1 cm^2	1 mm^2
Brilliance	10^{7}	10^{14}

Laser is an ideal light source in terms of brilliance!

Polarization of Light

- Three kinds of polarization states



- Stokes Parameters

$$s_0 = |\tilde{E}_x|^2 + |\tilde{E}_y|^2$$ Total Photon Flux

$$s_1 = |\tilde{E}_x|^2 - |\tilde{E}_y|^2$$ Horizontal-Vertical

$$s_2 = 2Re[\tilde{E}_x\tilde{E}_y{}^*]$$ 45ºlinear-135ºlinear

$$s_3 = 2Im[\tilde{E}_x\tilde{E}_y{}^*]$$ Right Hand-Left Hand

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

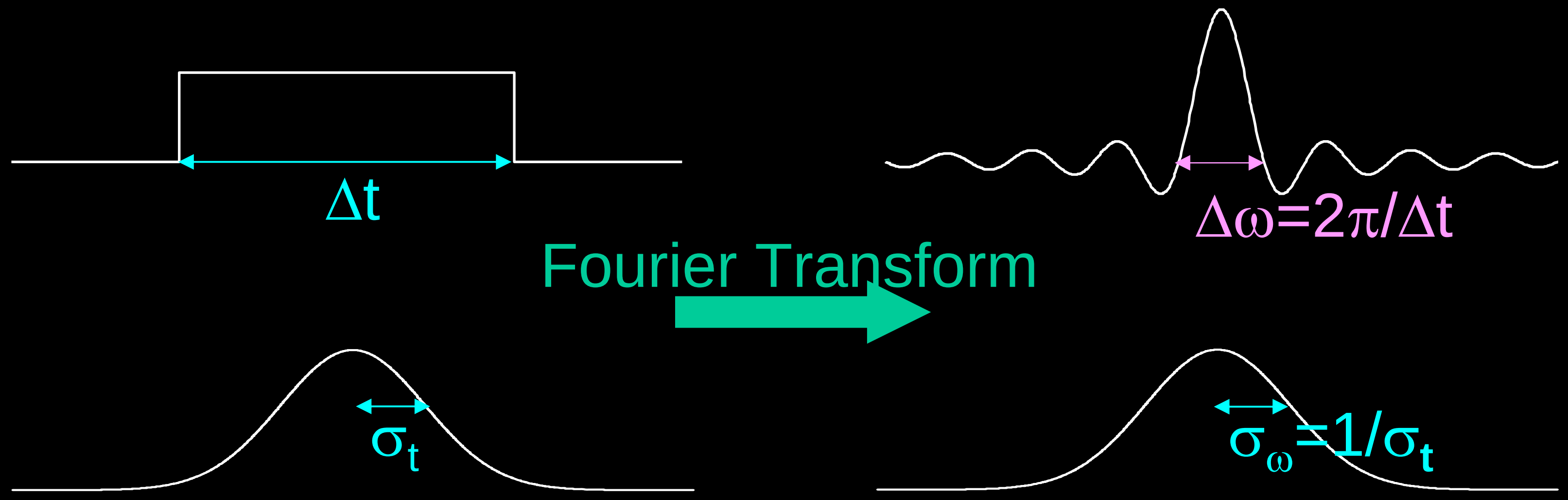
Uncertainty of Light

- The photon distribution in the 6D phase space (x,y,x',y',ω,t) gives us the full information on the properties of SR.
- Due to wave nature of light, however, we have two uncertainty relations to be concerned about, which are well characterized by the Fourier transform.
- These relations imposes two restrictions on SR, Fourier and Diffraction limits.

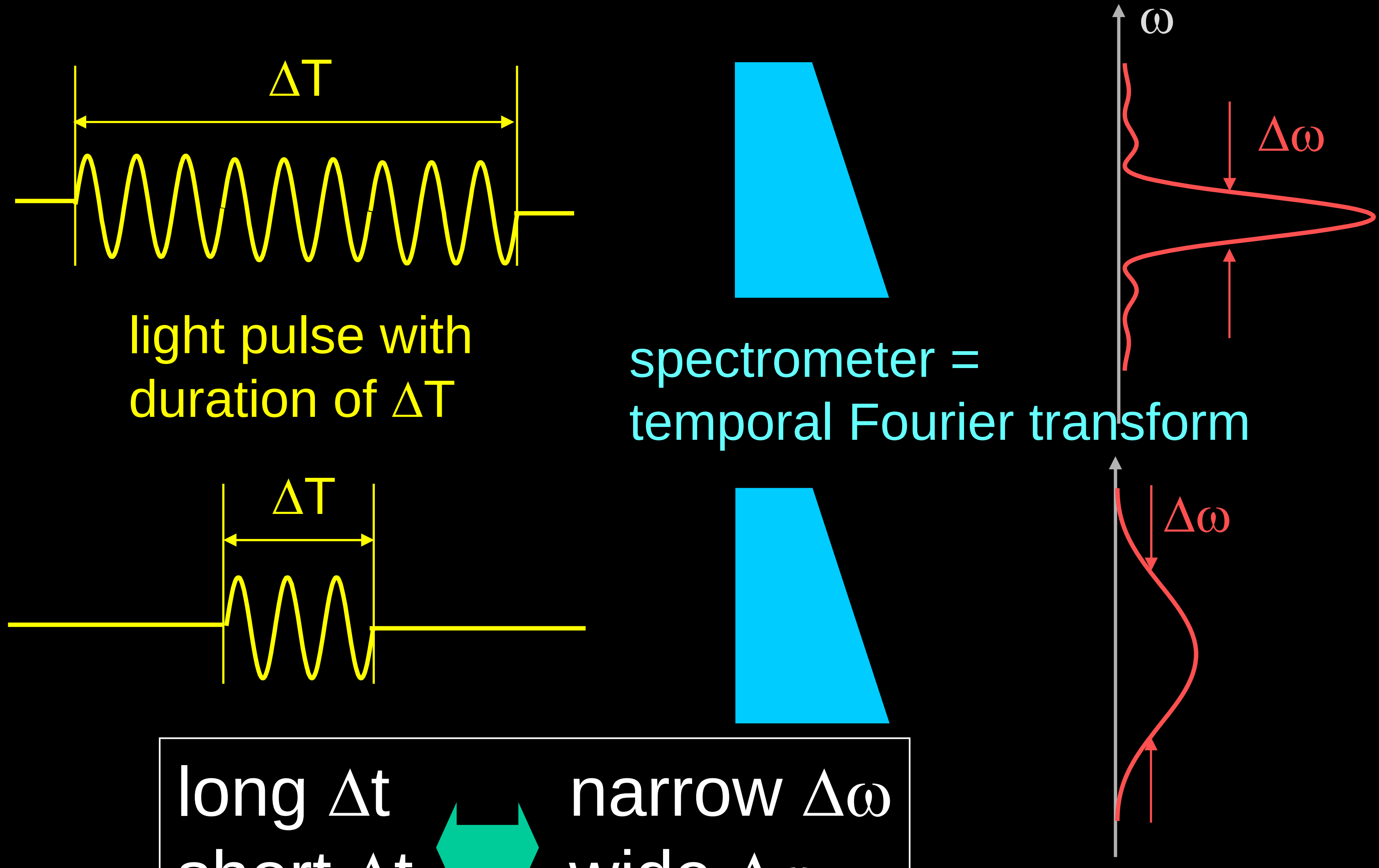
Fourier Transform: Example

Important Fourier Transform in SR Formulae

$F(t)$	$f(\omega) = \int_{-\infty}^{\infty} F(t) e^{i\omega t}$
$\begin{cases} 1/\Delta t; & -\Delta t/2 \leq t \leq \Delta t/2 \\ 0; & t < -\Delta t/2, \Delta t/2 < t \end{cases}$	$\dfrac{\sin \omega \Delta t/2}{\omega \Delta t/2} \equiv \mathrm{sinc}(\omega \Delta t/2)$
$\dfrac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\dfrac{t^2}{2\sigma_t^2}\right)$	$\exp\left(-\dfrac{\omega^2 \sigma_t^2}{2}\right)$



Temporal Fourier Transform



long Δt	⟷	narrow Δω
short Δt		wide Δω

Fourier Limit of Light

- Temporal Fourier transform imposes that the product of bandwidth and pulse duration of light is constant.

$$\Delta\omega \times \Delta t \sim \text{const.}$$

- Uncertainty of light in the (ω,t) plane.
- Important to understand the spectral functions of SR.

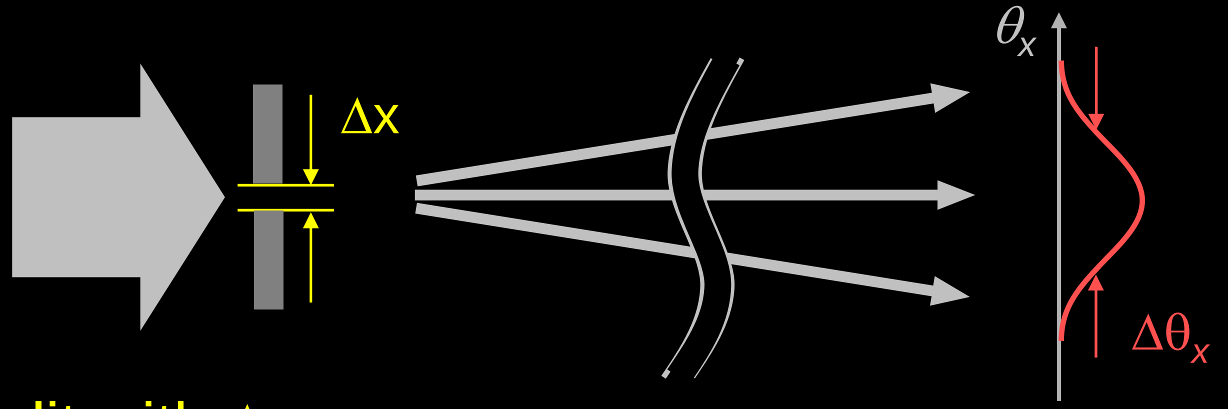
Monochromatic Light

- Ideally, "monochromatic" means

$$\Delta\omega = 0 \ \& \ \Delta t = \infty$$

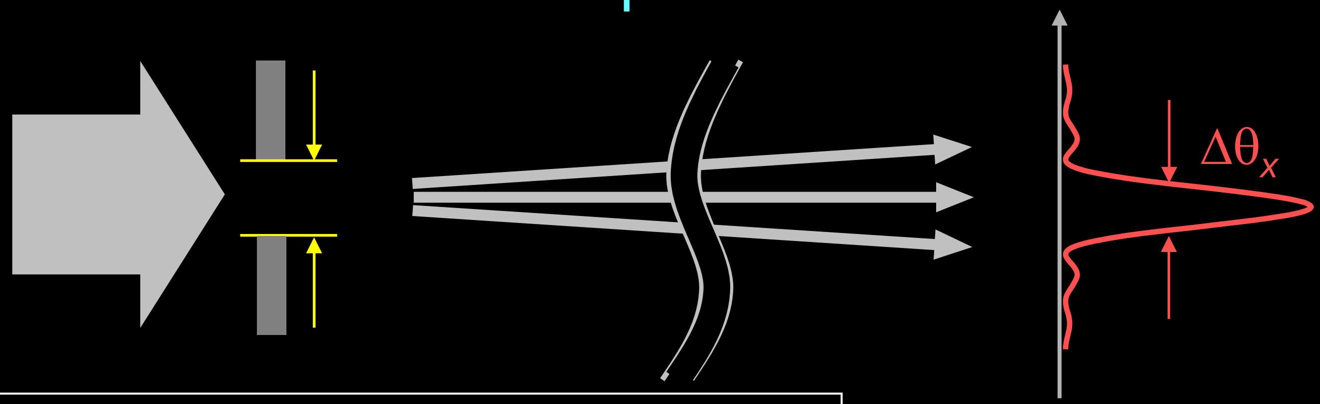
- In practice, perfectly monochromatic light does not exist, but all kinds of light are partially monochromatic.
- Temporal Fourier transform is to decompose the practical light into components of perfectly monochromatic light.

Spatial Fourier Transform



slit with Δx
= beam size

diffraction pattern in the far region
= spatial Fourier transform



wide Δx ↔ narrow $\Delta\theta_x$

narrow Δx ↔ wide $\Delta\theta_x$

Diffraction Limit of Light (1)

Photon Beam with 1D Gaussian Profile

$$|E(x)|^2 \propto \exp(-x^2/2\sigma_x^2)$$ Photon Spatial Profile

$$E(x) \propto \exp(-x^2/4\sigma_x^2)$$ E-field Spatial Profile

Fourier transform

$$\tilde{E}(k_x) \propto \exp(-k_x^2\sigma_x^2)$$ E-field k-vector Profile

$$|\tilde{E}(k_x)|^2 \propto \exp(-2\sigma_x^2 k_x^2)$$ Photon k-vector Profile

$$\equiv \exp(-\theta_x^2/2\sigma_{x'}^2)$$ Photon Angular Profile

$$\sigma_{x'}\sigma_x = \frac{\lambda}{4\pi}$$ Natural emittance of light with wavelength λ

Diffraction Limit of Light (2)

- Spatial Fourier transform imposes that the product of beam size and angular divergence scales as λ.

$$\sigma_{x,y} \times \sigma_{x',y'} \sim \lambda.$$

- Uncertainty of light in the (x,x',y,y') plane.
- Effective beam size and angular divergence are obtained by convolution with those of the electron beam.

Plane Wave

- Ideally, “plane wave” means

$$\sigma_{x',y'} = 0 \ \& \ \sigma_{x,y} = \infty$$

- In practice, perfect plane wave does not exist, but all kinds of light are partially plane-wave-like.
- Spatial Fourier transform is to decompose the practical light into components of perfect plane wave.

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

SR: Light from a Moving Electron

- Unlike the ordinary light (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different due to relativistic effects.
- What we have to take care is:
  1. Speed-of-light limit
  2. Squeezing of light pulse
  3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of an electron never exceeds the speed of light.

$$v/c = \beta \;=\; \sqrt{1-\gamma^{-2}}$$

$$\sim\; 1 - \frac{1}{2\gamma^2}$$

Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.999999998

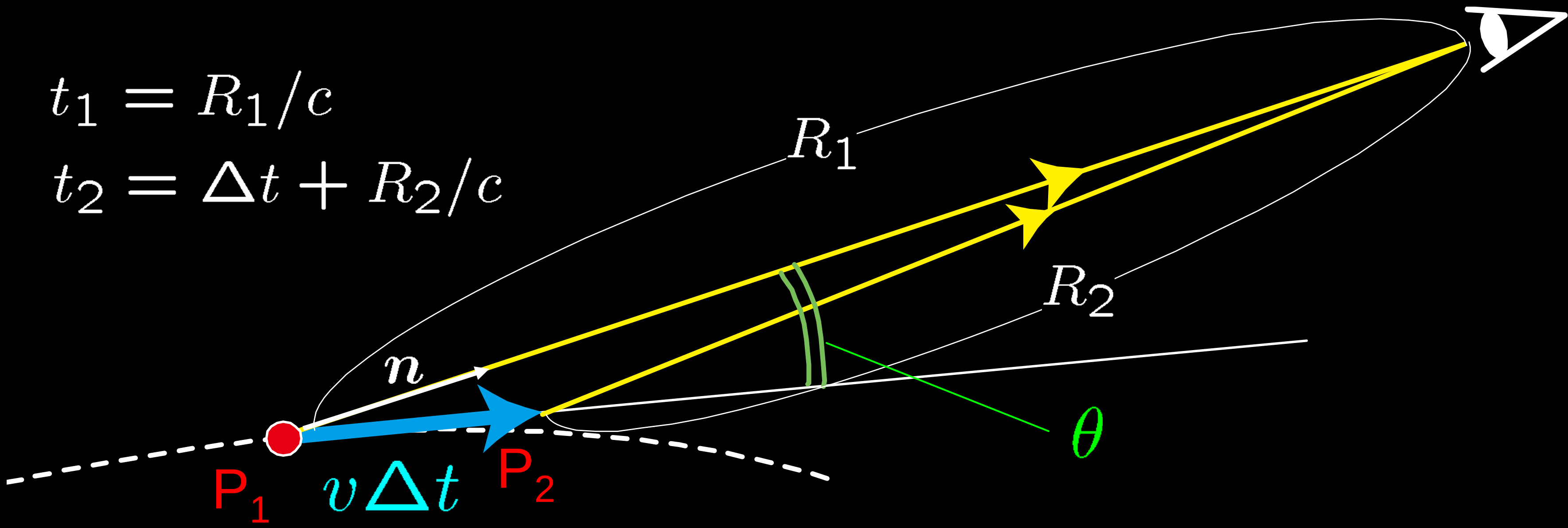
$$\gamma = \frac{E}{mc^2}$$

: Lorentz Factor (relative electron energy)

Squeezing of Light Pulse Duration

$$t_1 = R_1/c$$

$$t_2 = \Delta t + R_2/c$$



$$R_2 = \sqrt{(R_1)^2 + (v\Delta t)^2 - 2R_1 v\Delta t\cos\theta}$$

$$\sim R_1 - (\boldsymbol{v}\cdot\boldsymbol{n})\Delta t$$

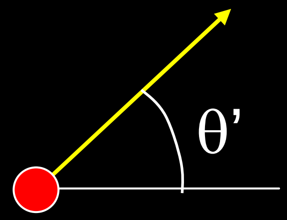
$$\Delta\tau = t_2 - t_1 = \Delta t + R_2/c - R_1/c$$

$$= \Delta t\,\boxed{(1-\boldsymbol{\beta}\cdot\boldsymbol{n})} = \boxed{\frac{\Delta t}{2\gamma^2}}$$

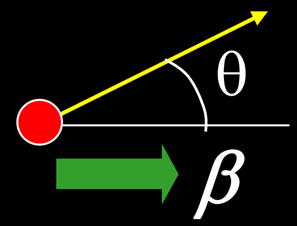
γ>>1,θ=0

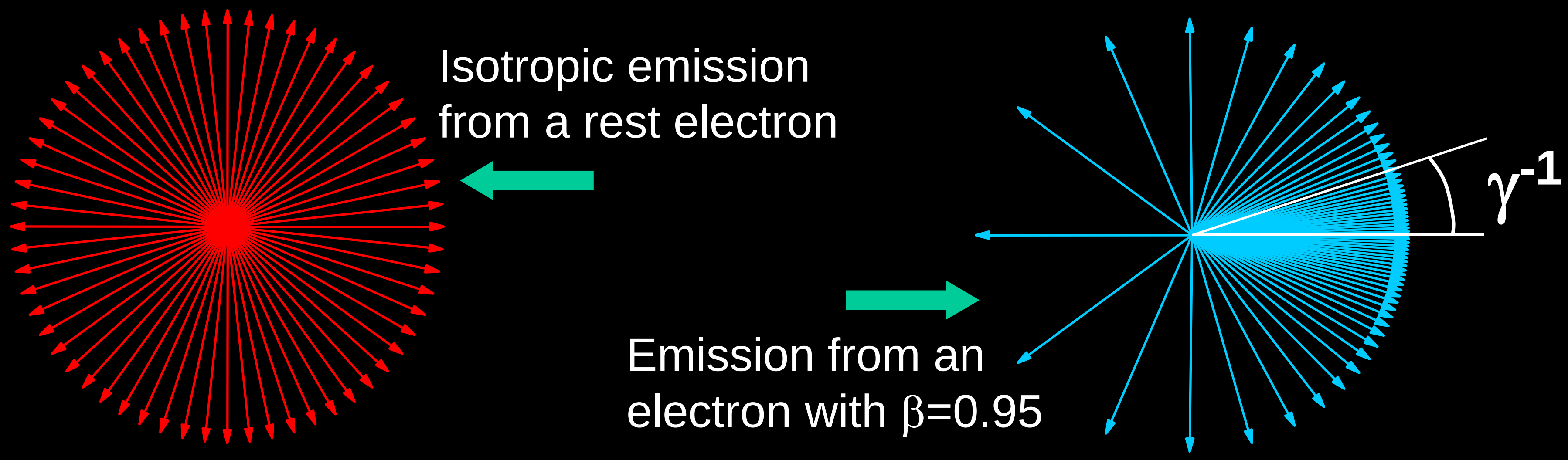
time squeezing

Conversion of Emission Angles



$$\theta = \tan^{-1}\left(\frac{\gamma^{-1}\sin\theta'}{\beta + \cos\theta'}\right)$$



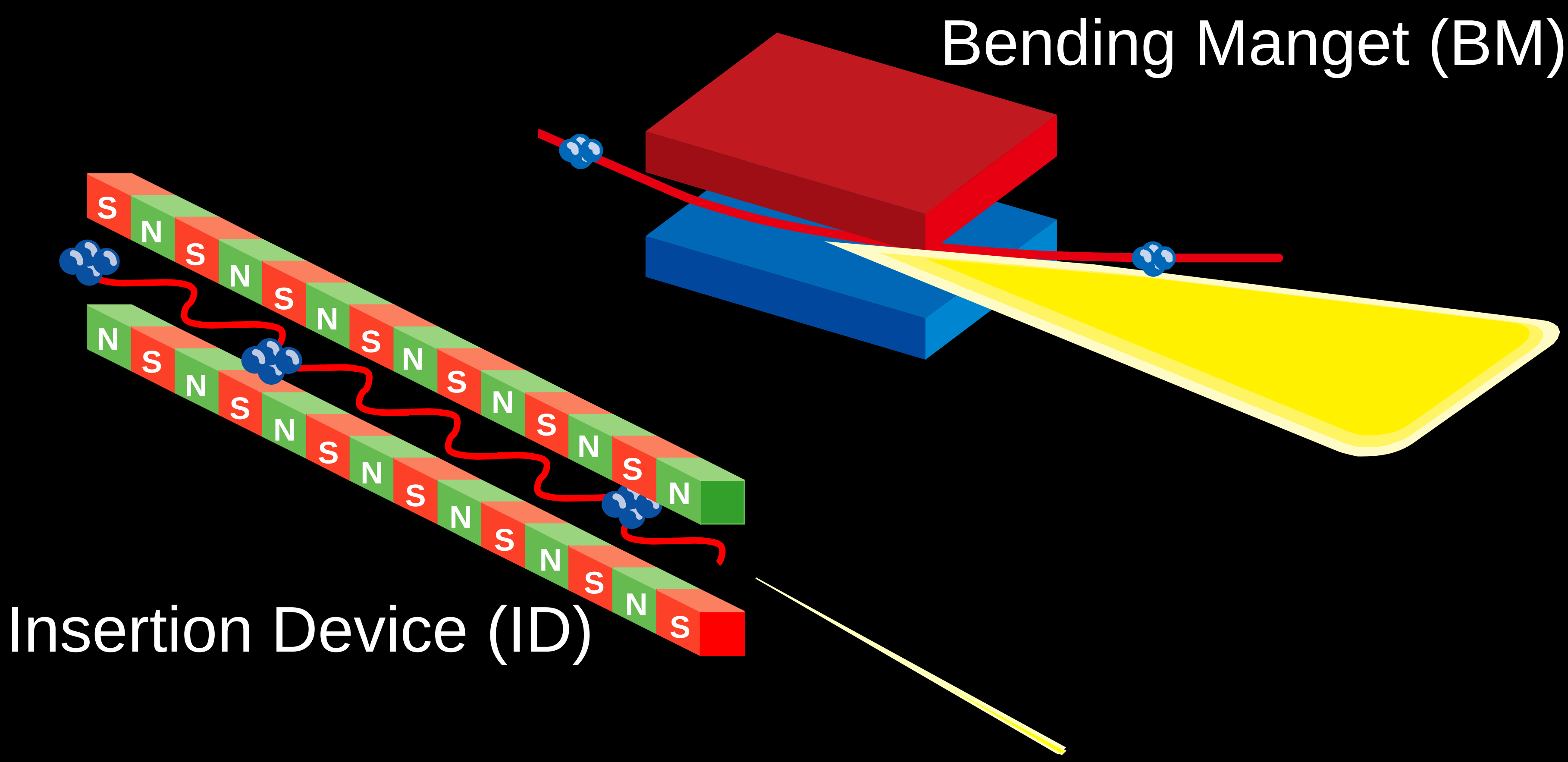


Light emitted from a moving object (β~1) concentrates within γ^{-1}

Overview of SR Light Source

What is SR Light Source?

Magnets to deflect the electron beam and generate SR.



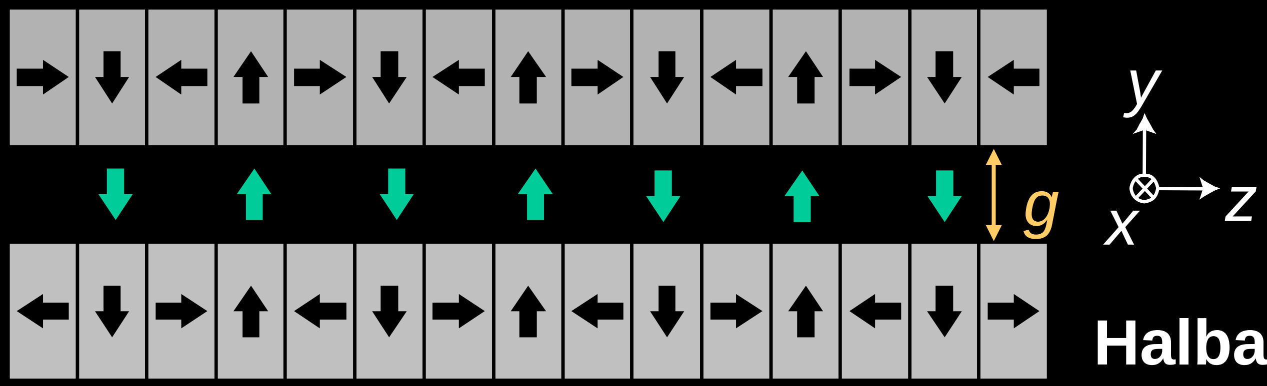
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate uniform field to guide the electron beam into a circular orbit.
- Electromagnets combined with highly-stable power supplies are adopted in most BMs due to stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

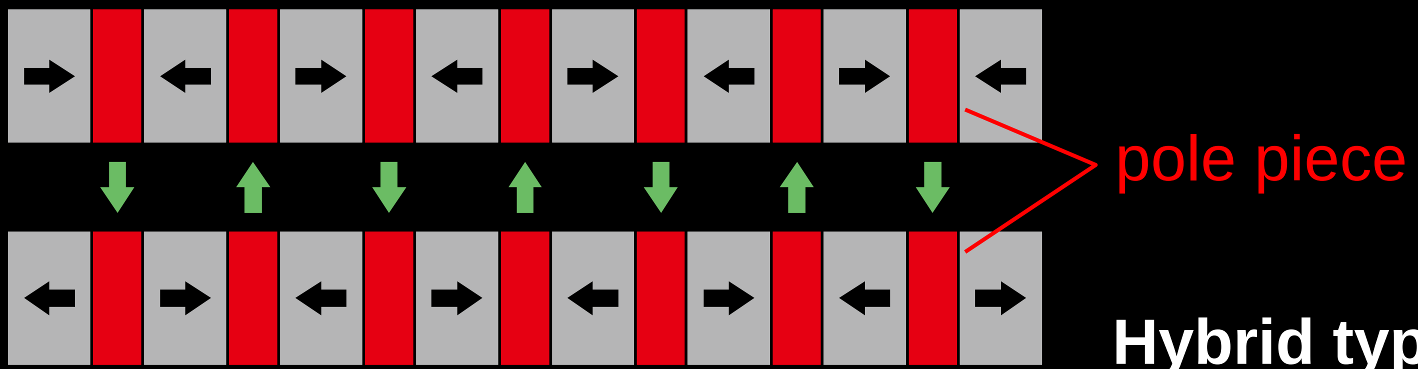
Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Classified into wigglers and undulators.

Magnetic Circuit of IDs



Halbach type

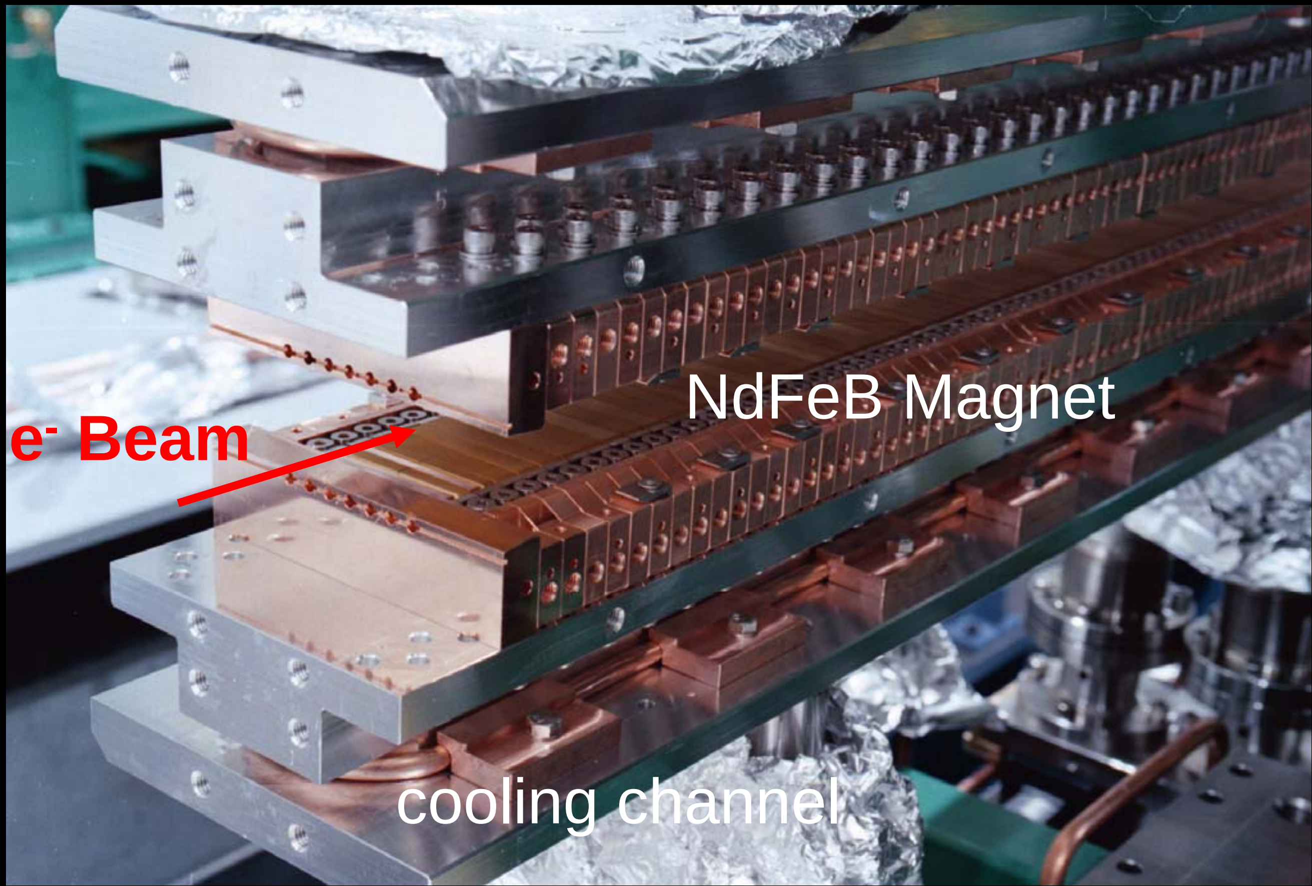


Hybrid type

In any type, a sinusoidal magnetic field is obtained: $B_y(z) \sim B_0(B_r, g/\lambda_u)\,\sin\left(\frac{2\pi z}{\lambda_u}\right)$

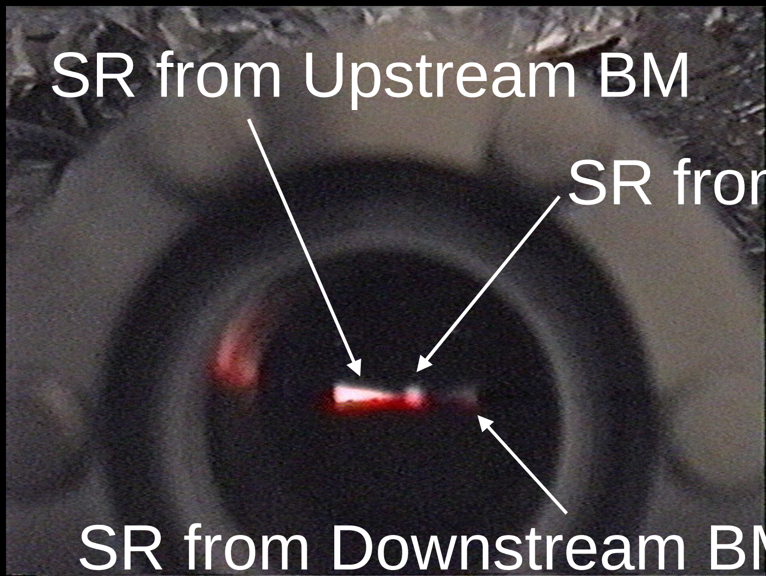
Example of ID Magnets

Halbach-type Magnet Array for SPring-8 Standard Undulators

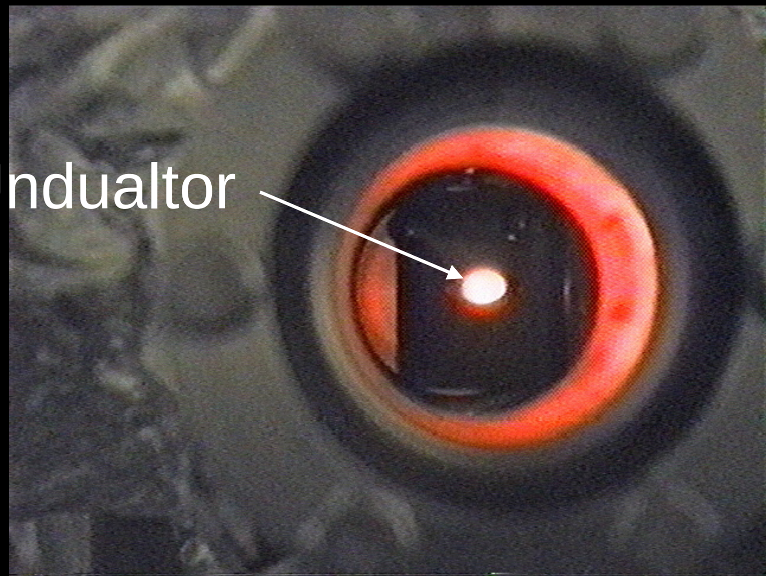


Example of SR Image

BL41XU@SP-8, First Image of SR
at Fluorescent Screen (<0.1mA)

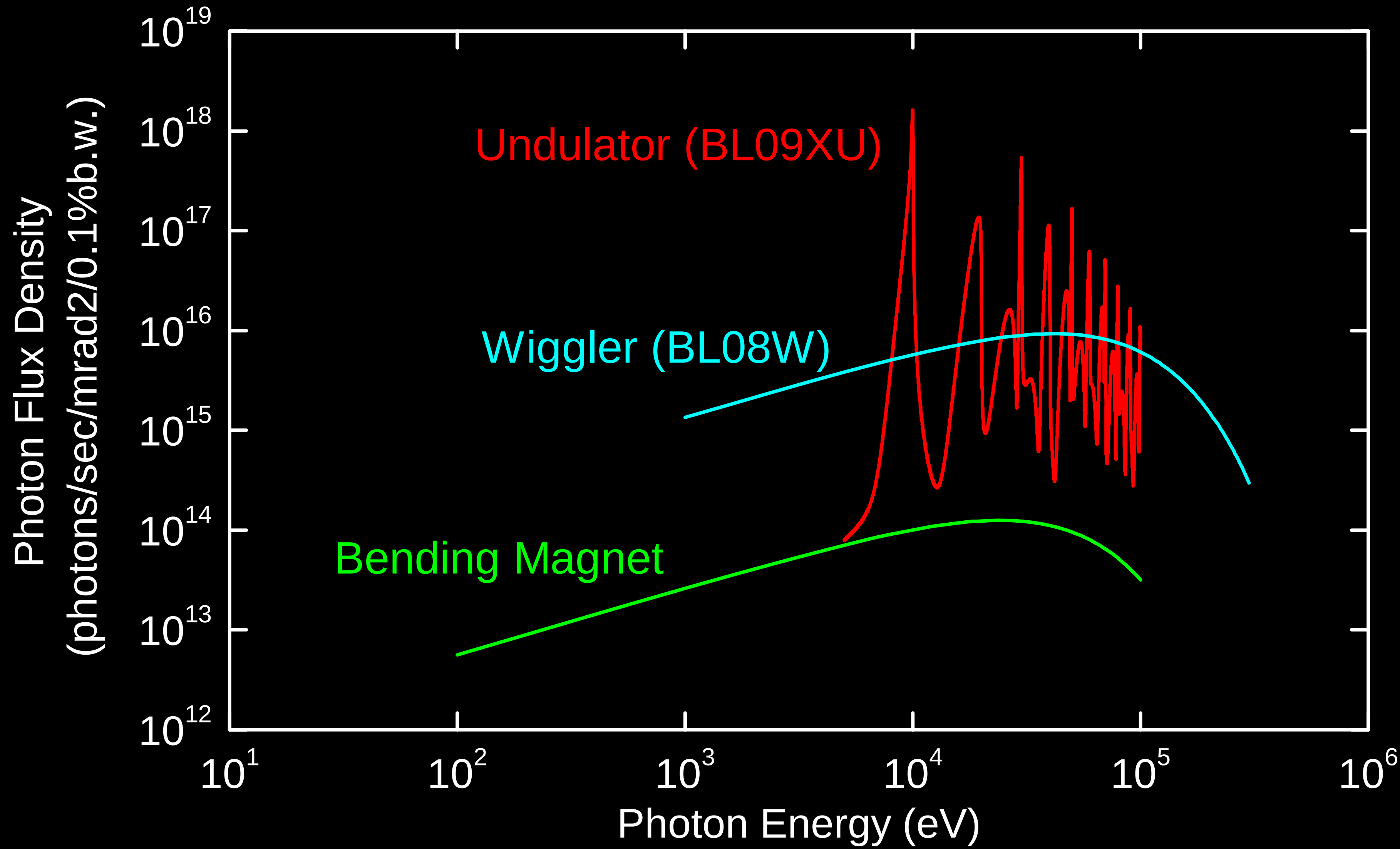


Undulator Gap = 50 mm



Undulator Gap = 20 mm

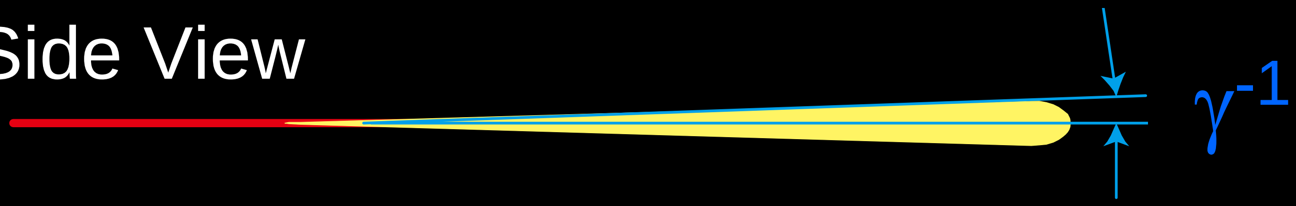
Comparison of Light Sources



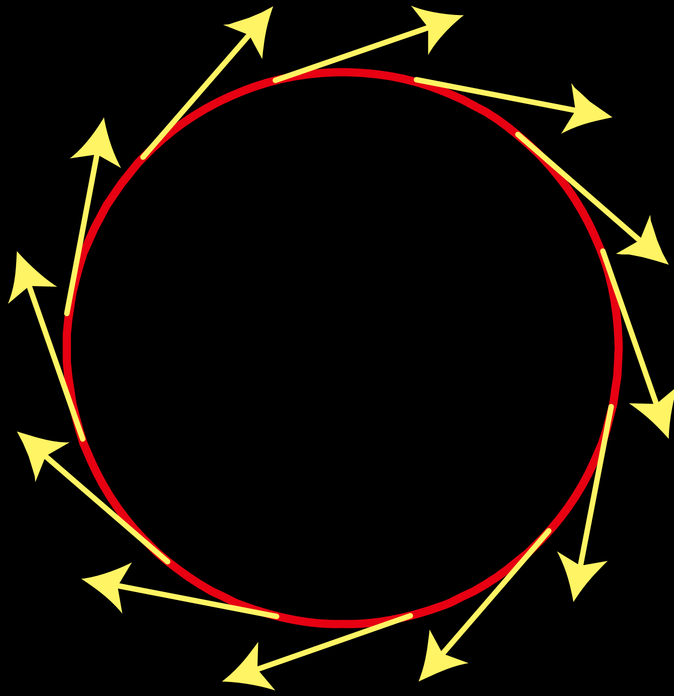
Characteristics of SR (1)
- Radiation from BMs

Directivity of BM Radiation

Side View



High directivity in the vertical plane
($\sigma_{y'} \sim \gamma^{-1} \sim 64\mu rad$@SP-8)

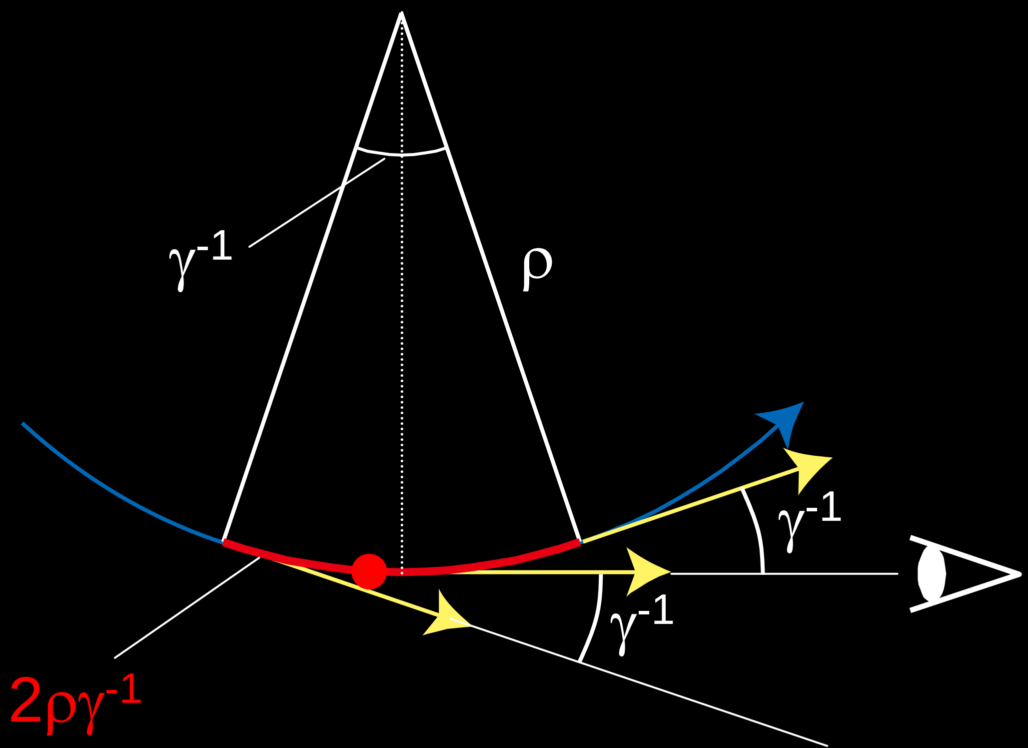


Isotropic in the orbitgal plane

Top View

2-dimensional directivity

Spectrum of BM Radiation (1)



Major contribution of radiation is from the portion painted red

Pulse duration for e⁻

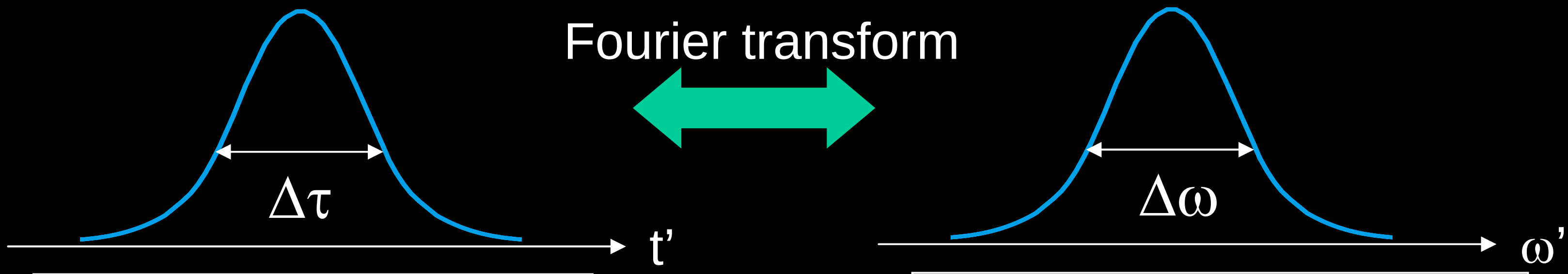
$$\Delta t = 2\rho\gamma^{-1}/c$$

squeezing

Pulse duration for observer

$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$

Spectrum of BM Radiation (2)



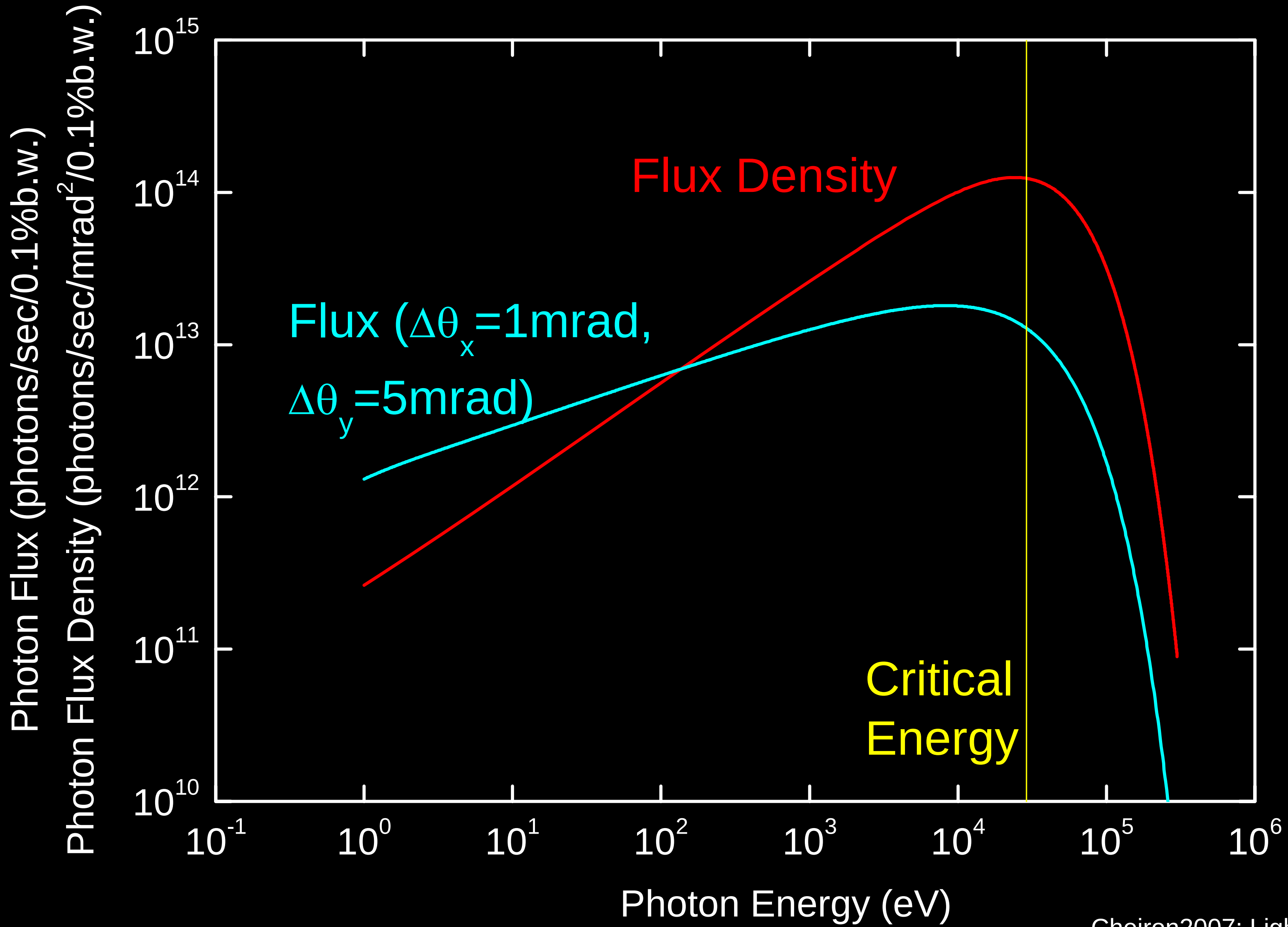
Light pulse with duration of $\Delta\tau$

White spectrum up to $\omega \sim \Delta\omega \sim 1/\Delta\tau$

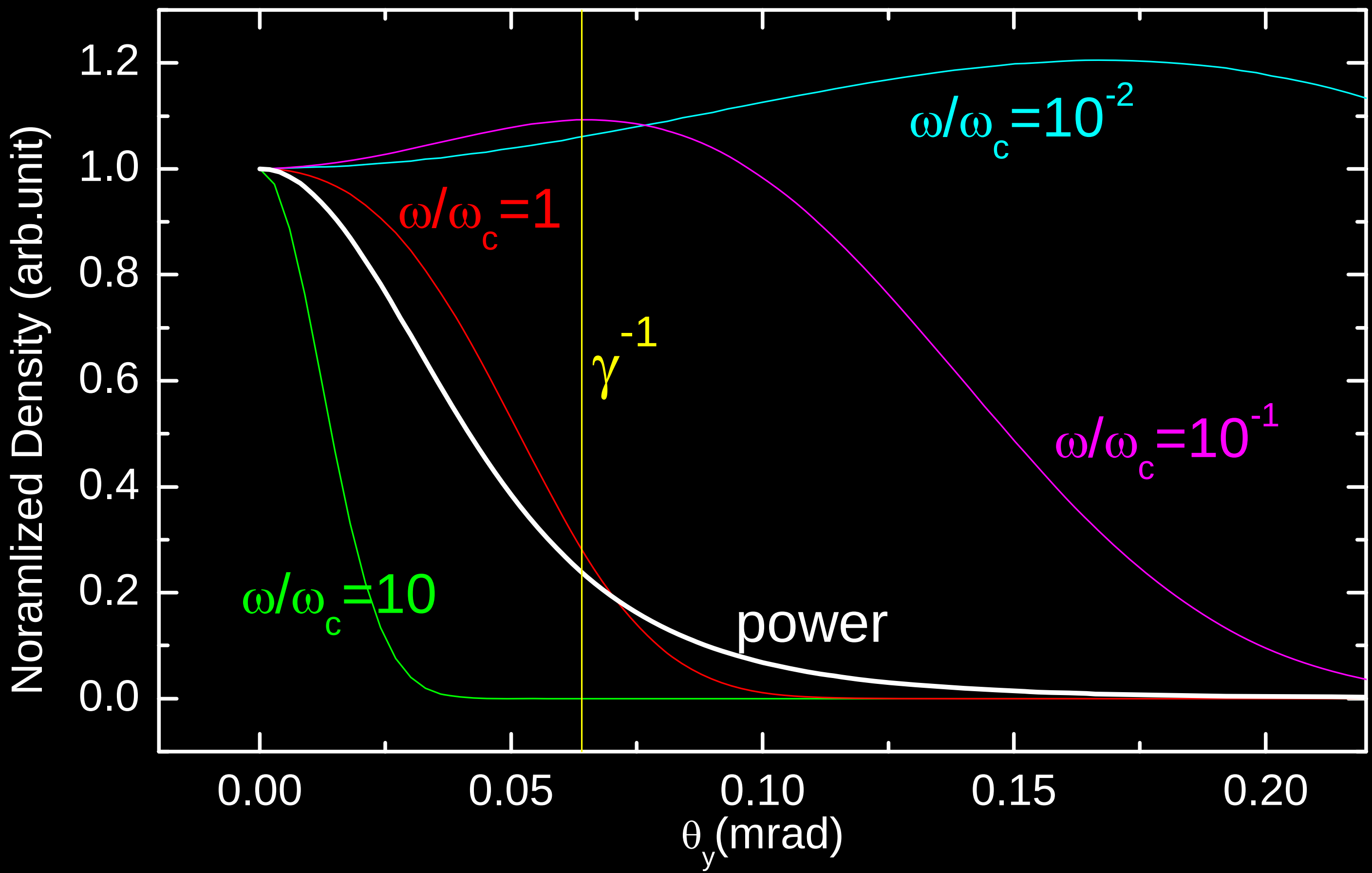
- By definition, $\omega_c = 3/2\Delta\tau = 3\gamma^3 c/2\rho$ is called "critical frequency" of SR, which gives a criterion of the maximum energy of SR from a BM.
- In practical units,

$$\hbar\omega_c(\mathrm{keV}) = 0.665 E_e^2(\mathrm{GeV}) B(\mathrm{T})$$

Example of Spectrum



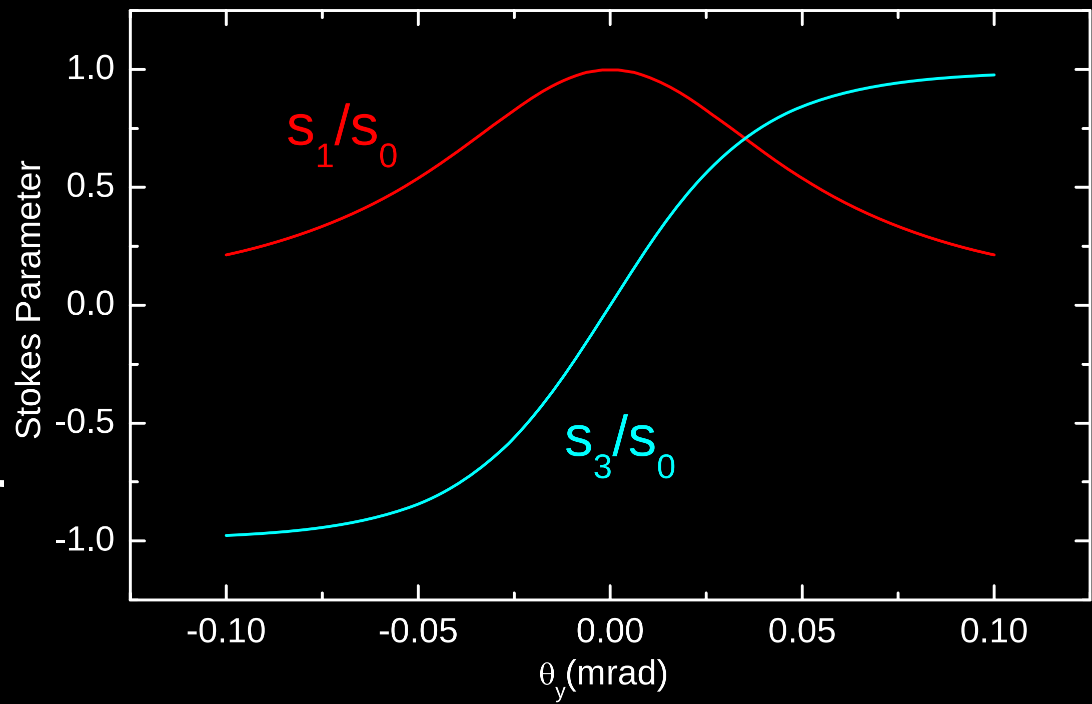
Angular Profile of BM Radiation

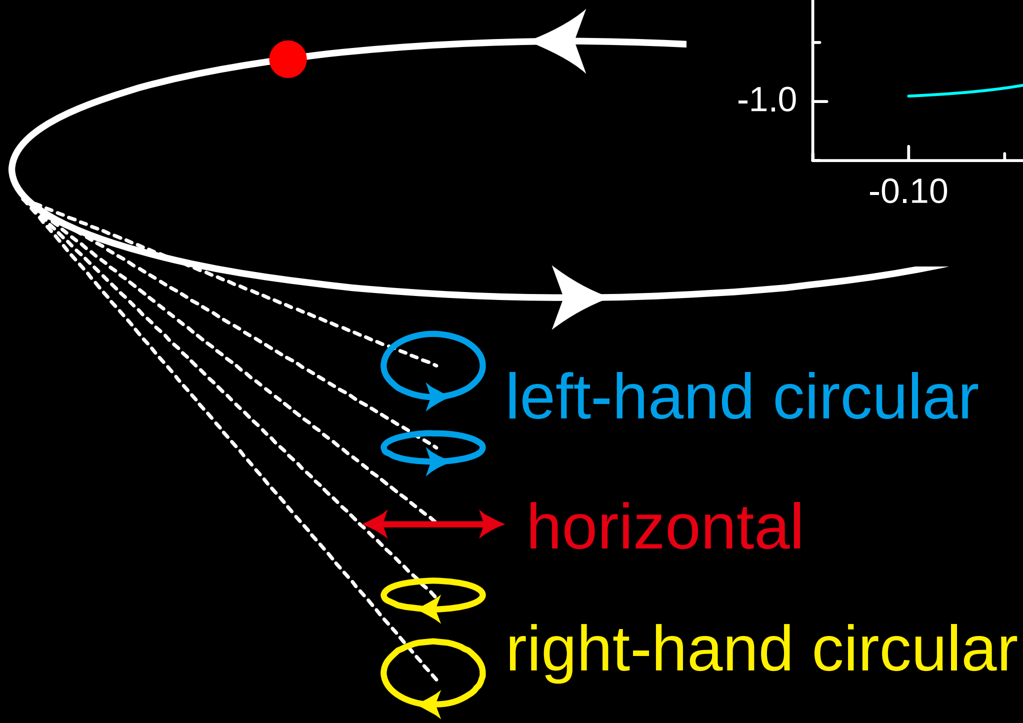


- power profile ~ flux profile@ω/ω_c=1
- larger angular spread for lower energy

Polarization of BM Radiation

Stokes parameters of BM radiation along vertical axis





Polarization state reflects the apparent motion of electron.